

Appendix A

Structure of reconstruction matrices

The motion-corrupted magnetization during the c^{th} readout is denoted $m_c(r) = |m_c(r)|e^{i\phi_c(r)}$.

- 5 The phase corruption during the c^{th} excitation is expressed by the $N^2 \times N^2$ matrix \mathbf{P}^c :

$$\mathbf{P}^c = \begin{pmatrix} e^{i\phi_c(r_1)} & 0 & \dots & 0 \\ 0 & e^{i\phi_c(r_2)} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & e^{i\phi_c(r_{N^2})} \end{pmatrix} \quad (7)$$

$\mathbf{P}^c \mathbf{m}$ is the phase-corrupted magnetization that is the source if signal during the c^{th} readout. Each of the C interleaves has a different phase corruption, represented by stacking the \mathbf{P}^c 's to form \mathbf{P} :

$$\mathbf{P} = \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \vdots \\ \mathbf{P}^C \end{pmatrix} \quad (8)$$

\mathbf{P} splits the magnetization vector \mathbf{m} into C separate phase-corrupted magnetization vectors. These vectors are stacked to form the $CN^2 \times 1$ vector \mathbf{Pm} representing the phase-corrupted magnetization that is sampled over the course of the experiment.

- 15 Each of the phase-corrupted magnetization vectors is separately Fourier transformed by the matrix \mathbf{F} ($CN^2 \times CN^2$):

$$\mathbf{F} = \begin{pmatrix} F^N & 0 & \dots & 0 \\ 0 & F^N & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & F^N \end{pmatrix} \quad (9)$$

\mathbf{F} is composed of C 2DFT matrices \mathbf{F}^N ($N^2 \times N^2$) consisting of the Fourier kernels:

$$\mathbf{F}_{lm}^N = e^{i2\pi k_l \cdot r_m} \quad (10)$$

- 20 The product \mathbf{FPm} is the $CN^2 \times 1$ vector representing the C phase-corrupted spectra, sampled in Cartesian coordinates.

Finally, the spectra given by \mathbf{FPm} are resampled onto the k -space sampling trajectories by the resampling matrix \mathbf{G} ($CR \times CN^2$):

$$\mathbf{G} = \begin{pmatrix} G^1 & 0 & \dots & 0 \\ 0 & G^2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & G^c \end{pmatrix} \quad (11)$$

Each submatrix \mathbf{G}^c ($R \times N^2$) interpolates the Cartesian k -space points \mathbf{k}_l onto the sampling points for the c^{th} readout s_m^c :

$$\mathbf{G}_{lm}^c = \text{sinc}(\mathbf{k}_l - s_m^c) \quad (12)$$

- 5 Note that for trajectories that collect data on Cartesian coordinates (such as spinwarp or interleaved EPI), the elements of \mathbf{G} are discrete delta functions.

APPENDIX B

The mixing matrix

Using the notation of Appendix A, the elements of the mixing matrix M for an arbitrary trajectory are:

$$5 \quad M_{lm} = \sum_{c=1}^C e^{i(\phi_c(r_m) - \phi_c(r_l))} \sum_{r=1}^R \left[\sum_{n=1}^{N^2} \sin c(k_n - s_r^c) e^{+i2\pi k_n \cdot r_m} \right] \left[\sum_{n=1}^{N^2} \sin c(k_n - s_r^c) e^{-i2\pi k_n \cdot r_l} \right] \quad (13)$$

The summation over each readout r described non-idealities in the k -space trajectory that causes coupling between voxels within a readout. The phase difference describes the aliasing caused by motion-induced phase errors. These phase offsets are manifested as perturbations of the encoding functions used in the trajectory.